MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2016/2017**

## DIM5058 – MATHEMATICAL TECHNIQUES 1

( T71, 8082 )

25 FEBRUARY 2017

2.30 pm – 4.30 pm

( 2 Hours )

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**INSTRUCTIONS TO STUDENT**

1. This question paper consists of **2** pages excluding cover page and appendix.
2. Attempt **ALL FOUR (4)** questions. The marks distribution for each question is given.
3. Write your answers in the answer booklet provided.
4. Key formulae are given in the Appendix.

## Question 1

a) Solve for $x$ by factoring the equation $2x^3 - 18x = 3x^2 - 27$. (6 marks)

b) Molly practices shooting a free throw on a basketball court. She succeeds to make 78% of her shots. How many shots does she miss if 39 of her shots are successful? (3 marks)

c) Solve the following inequality, express your answer in **interval notation** and sketch the solution on a real **number line**.

i. $x - 1 \geq \dfrac{(8x+2) - 2x}{3}$ (5 marks)

ii. $|2(x-5)| - 16 + 2x < 2x$ (6 marks)

**[TOTAL 20 MARKS]**

## Question 2

a) Let $f(x) = \begin{cases} x - \dfrac{7}{3} & \text{if } x < 3 \\ \sqrt{2x^2 + 4} & \text{if } x \geq 3 \end{cases}$, find $f(4)$. (2 marks)

b) Given that $f(x) = 3 - \dfrac{2}{x}$ and $g(x) = \dfrac{5}{x} + 1$.

i. Find $\left(\dfrac{f}{g}\right)(x)$ and simplify the answer. (5 marks)

ii. Calculate $\left(\dfrac{f}{g}\right)\left(\dfrac{1}{2}\right)$. (2 marks)

c) Prove that function $f(x) = 3x^5 - 4x^3 + \dfrac{2}{3}x$ is an odd function. (3 marks)

d) Given a quadratic function $f(x) = x^2 + 6x - 7$.

i. Determine whether the parabola opens upward or downward. (1 mark)
ii. Find the vertex of the parabola. (3 marks)
iii. Find the $x$-intercept(s). (3 marks)
iv. Find the $y$-intercept. (2 marks)
v. Sketch the parabola and label the necessary points. (4 marks)

**[TOTAL 25 MARKS]**

**Continued...**

## Question 3

a) Find the value of $x$ and $y$ if $\begin{bmatrix} 7 & 2x+1 & -6 \\ -2 & 3 & y-4 \end{bmatrix} - \begin{bmatrix} -4 & 5 & 9 \\ -1 & 2 & -3y \end{bmatrix} = \begin{bmatrix} 11 & 6 & -15 \\ -1 & 1 & -20 \end{bmatrix}$. (3 marks)

b) Find the values of $m$ for which the matrix is **not invertible**. (6 marks)

$$\begin{bmatrix} 4 & m+1 \\ m-2 & 1 \end{bmatrix}$$

c) Given matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 1 & 0 \\ 2 & 5 & 1 \end{bmatrix}$.

i. Calculate the determinant, $|A|$. (2 marks)

ii. Find the adjoint of $A$. (6 marks)

iii. From answer in part (c)i and part (c)ii, state the $A^{-1}$. (1 mark)

d) The table below shows the result on "The Programmer Competition" where the participants need to go through 2 different rounds in the duration of time (in minutes).

| | Team $X$ | Team $Y$ |
|---|---|---|
| Round 1 | 256 | 320 |
| Round 2 | 650 | 375 |

i. If the prizes for Round 1 and Round 2 is RM3200 and RM5500 respectively, represent the information in the form of $AX=B$. (2 marks)
*[Hint: let x and y be the number of participants from Team X and Team Y, respectively]*

ii. By using the *Cramer's Rule*, find the number of participants from each team. (10 marks)

**[TOTAL: 30 MARKS]**

## Question 4

a) Write the first five terms of the sequence $a_n = \dfrac{(-1)^{n+1}}{(x-n)^3}$. (5 marks)

b) The third term and fifth term of a geometric progression are 80 and 1280 respectively. If all the terms are positive, find the

i. common ratio and the first term. (5 marks)

ii. sum of the first seven terms. (2 marks)

c) ToshZ company sells computer appliances. As a commission, his staff will receive RM170 for the first computer sold, RM200 for the second computer sold, RM230 for the third computer sold and so on.

i. Find the commission received (in RM) on the seventh ($7^{th}$) computer sold. (4 marks)

ii. How many computer has he sold if the commission received is RM440? (5 marks)

d) Find the expansion of $\left(y + \dfrac{2}{y}\right)^3$ by using the Binomial Theorem. (4 marks)

**[TOTAL: 25 MARKS]**

**End of page.**

# APPENDIX

| Inequality | Solution |
|---|---|
| $\|x\| < a$ | $-a < x < a$ |
| $\|x\| \le a$ | $-a \le x \le a$ |
| $\|x\| > a$ | $x < -a$ or $x > a$ |
| $\|x\| \ge a$ | $x \le -a$ or $x \ge a$ |

***Completing the square:*** $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

***Quadratic formula:*** If $ax^2 + bx + c = 0$ where $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

***Standard form of a quadratic function:*** $f(x) = a(x - h)^2 + k$, $a \neq 0$

| *Determinant of a 2 × 2 matrix* | *Determinant of a 3 × 3 matrix* |
|---|---|
| $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - a_2 b_1$ | $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ $= a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ |
| ***Inverse of a 2 × 2 matrix*** | ***Inverse of a 3 × 3 matrix*** |
| If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $ad - bc \neq 0$. | $A^{-1} = \dfrac{1}{\|A\|}\left[c_{ij}\right]^T$ $A^{-1} = \dfrac{1}{\|A\|}\begin{bmatrix} c_{11} & c_{21} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{23} & c_{33} \end{bmatrix}$ $A^{-1} = \dfrac{1}{\|A\|} adj\ A$ where $\left[c_{ij}\right]^T$ is called the adjoint of A (adj A). $c_{ij}$ of the entry $a_{ij} = (-1)^{i+j} M_{ij}$ |

| *Cramer's Rule for 2 × 2 matrix* | *Cramer's Rule for 3 × 3 matrix* |
|---|---|
| If $\begin{aligned} a_1x+b_1y&=c_1\\ a_2x+b_2y&=c_2 \end{aligned}$<br>then $x=\dfrac{\begin{vmatrix} c_1 & b_1\\ c_2 & b_2\end{vmatrix}}{\begin{vmatrix} a_1 & b_1\\ a_2 & b_2\end{vmatrix}}$ and $y=\dfrac{\begin{vmatrix} a_1 & c_1\\ a_2 & c_2\end{vmatrix}}{\begin{vmatrix} a_1 & b_1\\ a_2 & b_2\end{vmatrix}}$ where<br>$\begin{vmatrix} a_1 & b_1\\ a_2 & b_2\end{vmatrix}\neq 0$ | $\begin{aligned} a_1x+b_1y+c_1z&=d_1\\ \text{If } a_2x+b_2y+c_2z&=d_2\\ a_3x+b_3y+c_3z&=d_3 \end{aligned}$<br>then $x=\dfrac{D_x}{D}, y=\dfrac{D_y}{D}, z=\dfrac{D_z}{D}$ where<br>$D=\begin{vmatrix} a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{vmatrix}$, $D_x=\begin{vmatrix} d_1 & b_1 & c_1\\ d_2 & b_2 & c_2\\ d_3 & b_3 & c_3\end{vmatrix}$,<br>$D_y=\begin{vmatrix} a_1 & d_1 & c_1\\ a_2 & d_2 & c_2\\ a_3 & d_3 & c_3\end{vmatrix}$, $D_z=\begin{vmatrix} a_1 & b_1 & d_1\\ a_2 & b_2 & d_2\\ a_3 & b_3 & d_3\end{vmatrix}$ |

| *Arithmetic sequence* | *Geometric sequence* |
|---|---|
| $a_n=a_1+(n-1)d$ | $a_n=a_1r^{n-1}, S_n=\dfrac{a_1(1-r^n)}{1-r}$ |
| $S_n=\dfrac{n}{2}(a_1+a_n)$ | $S_\infty=\dfrac{a_1}{1-r}, \lvert r\rvert<1$ |

***Binomial Theorem***

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k;\quad n\geq 1$$

The $(r+1)^{st}$ term in the expansion of $(a+b)^n$ is $\binom{n}{r}a^{n-r}b^r$.